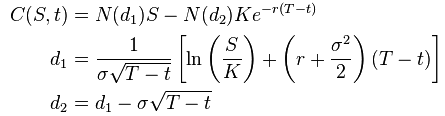
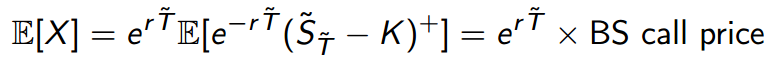
**Monte Carlo Simulation Project Report**

**Pricing Theory:**

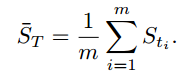
For European Vanilla Call, we would usually use Black-Sholes model:



For the Asian call:



where:



Combined the theories above with Monte Carlo simulation using different methods, we could develop Asian call option pricing program as requested.

For all programming below, always use box-muller for the picking of RV’s, for based on experience, this RV generating method is the fastest.

# **Part I: Monte Carlo Simulation on Asian call**

For this part, the Asian Call is defined by the following parameters:

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Strike price K = 100;

Time to Maturity T =1;

Current asset price is S0 = 100;

Risk free interest rate is r = 10%;

volatility of the asset per year is = 20%;

Discrete monitoring (m = 50);

The pricing results with and without control Variate are as followed:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Sample Size** | **Without Control Variate** | | | | **With Control Variate** | | | |
| **Price** | **Standard Error** | **Time** | **Efficiency** | **Price** | **Standard Error** | **Time** | **Efficiency** |
| 1000 | 6.95639 | 0.2931473 | 0.009 | 0.000773418 | 7.157641 | 0.00889385 | 0.01 | 7.91006E-07 |
| 10000 | 7.14586 | 0.0945359 | 0.053 | 0.000473663 | 7.164526 | 0.00298404 | 0.07 | 6.23315E-07 |
| 100000 | 7.15309 | 0.0302455 | 0.567 | 0.000518686 | 7.164458 | 0.000889567 | 0.759 | 6.00619E-07 |
| 300000 | 7.15317 | 0.0175453 | 1.543 | 0.000474993 | 7.164856 | 0.000518469 | 2.114 | 5.68265E-07 |
| 500000 | 7.15456 | 0.0134563 | 2.398 | 0.000434211 | 7.164579 | 0.000413218 | 3.549 | 6.05989E-07 |
| 800000 | 7.16357 | 0.0107465 | 3.758 | 0.000434001 | 7.164466 | 0.000324792 | 5.349 | 5.64265E-07 |
| 1000000 | 7.16047 | 0.0095685 | 4.572 | 0.000418595 | 7.164457 | 0.000282192 | 6.598 | 5.25414E-07 |
| 10000000 | 7.16015 | 0.0007685 | 13.278 | 7.84188E-06 | 7.164435 | 0.000109835 | 15.561 | 1.87722E-07 |

Based on the table shown above, we could tell that in order to get an accurate result, we should increase the sample size. And when the sample size reaches 1000000, the estimated price tends to be stable while rounding to the second decimal, which gives an estimated price of 7.16.

If we only look at the time difference, we might think that the method without control variate costs less. However, combined standard error, the comparison would give us different conclusion. And in fact, standard error is quite important for the accuracy.

While comparing the efficiency of the two method, we could tell that from the chart above, that as long as the sample size is above 1000, with control variate, we would always have a higher efficiency. With increasing sample size, the efficiency of Monte Carlo simulation with control variate tends to converge much faster to a stable level compared with the simulation without control variate. Thus, in this case, Monte Carlo Simulation with control variate would be a better choice.

# **Part II: Quasi-Monte Carlo Simulation on Asian call**

**Comparison between N and L in Quasi-Monte Carlo with control variate:**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **N\*M** | **N= 1000000** | | | | **L=10** | | | |
| **Price** | **Standard Error** | **Time** | **Efficiency** | **Price** | **Standard Error** | **Time** | **Efficiency** |
| 100000 | 7.16371 | 0.000894244 | 0.775302 | 6.20E-07 | 7.16489 | 0.00286144 | 0.732829 | 6.00E-06 |
| 300000 | 7.16443 | 0.00089708 | 2.21946 | 1.79E-06 | 7.16461 | 0.00164269 | 2.09451 | 5.65E-06 |
| 500000 | 7.16468 | 0.000899035 | 3.52302 | 2.85E-06 | 7.16435 | 0.00126835 | 3.52104 | 5.66E-06 |
| 800000 | 7.16454 | 0.000897845 | 5.65715 | 4.56E-06 | 7.16468 | 0.00100478 | 5.48808 | 5.54E-06 |
| 1000000 | 7.16467 | 0.000898231 | 7.00138 | 5.65E-06 | 7.16467 | 0.000898231 | 6.8598 | 5.53E-06 |

Plot for comparison of SE for both method:

Although from the chart above, we could tell the difference between the efficiency. However, based on the trend above, we could tell that with fixed N, SE tends to be stable. However, with fixed L, SE tends to show huge drop. Thus, we may choose fixed L for the following comparison in order to get much better accuracy.

**Quasi-Monte Carlo using same parameter from Part I with fixed L=10:**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Sample Size** | **Without control variate** | | | | **With control variate** | | | |
| **Price** | **Standard Error** | **Time** | **Efficiency** | **Price** | **Standard Error** | **Time** | **Efficiency** |
| 10000 | 7.13878 | 0.3069779 | 0.081033 | 0.467559 | 7.16489 | 0.00286144 | 0.732829 | 6.00E-06 |
| 100000 | 7.2085 | 0.09663565 | 0.85373 | 0.149388 | 7.16461 | 0.00164269 | 2.09451 | 5.65E-06 |
| 500000 | 7.1723 | 0.04296762 | 4.04348 | 0.065176 | 7.16435 | 0.00126835 | 3.52104 | 5.66E-06 |
| 800000 | 7.16758 | 0.03317666 | 6.40089 | 0.051801 | 7.16468 | 0.00100478 | 5.48808 | 5.54E-06 |
| 1000000 | 7.16989 | 0.03063656 | 8.46053 | 0.046844 | 7.16467 | 0.000898231 | 6.8598 | 5.53E-06 |

Here, from the chart above, similar as what we have concluded in Part I, the Quasi-Monte Carlo simulation with control variate shows lower SE and higher efficiency.

**Comparison between Monte-Carlo Simulation in Part I and Quasi-Monte Carlo Simulation with control variate**:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Sample Size** | **Price** | | **Standard Error** | | **Efficiency** | |
| **Part I** | **Quasi** | **Part I** | **Quasi** | **Normal** | **Quasi** |
| **10000** | 7.164458 | 7.16435 | 0.000889567 | 0.00126835 | 6.00619E-07 | 0.00126835 |
| **100000** | 7.164856 | 7.16459 | 0.000518469 | 0.0011591 | 5.68265E-07 | 0.0011591 |
| **500000** | 7.164579 | 7.16435 | 0.000413218 | 0.00107377 | 6.05989E-07 | 0.00107377 |
| **800000** | 7.164466 | 7.16468 | 0.000324792 | 0.00100478 | 5.64265E-07 | 0.00100478 |
| **900000** | 7.164457 | 7.16431 | 0.000282192 | 0.000946165 | 5.25414E-07 | 0.00094617 |
| **1000000** | 7.164435 | 7.16467 | 0.000109835 | 0.000898231 | 1.87722E-07 | 0.00089823 |

Here, as the chart above, we could tell the normal Monte Carlo has a better efficiency with control variate, whiled compared with the Quasi-Monte Carlo with control variate. Also, in this case, we could see that normal Monte-Carlo simulation with control variate has smaller SE, which indicates higher accuracy. Thus, in this case, normal monte carlo with control variate seems to be a better choice for option pricing.

# **Part III: Asian call with continuous average**

Here, the given parameters are as followed:

Strike price K = 2;

Time to Maturity T =2;

Current asset price is S0 = 2;

Risk free interest rate is r = 0.05;

volatility of the asset per year is = 0.5;

**Comparison between Normal Monte Carlo with control variate and randomized Quasi-Monte Carlo both under 10000000 simulations :**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Dimension**  **m** | **Normal MC with control variate** | | | | **Randomized Quasi-MC** | | | |
| **Price** | **Standard Error** | **Time** | **Efficiency** | **Price** | **Standard Error** | **Time** | **Efficiency** |
| 50 | 0.3553 | 0.00033 | 0.673 | 7.33E-08 | 0.3562 | 0.001079 | 0.809 | 9.406E-07 |
| 100 | 0.3524 | 0.000339 | 1.358 | 1.561E-07 | 0.353 | 0.001055 | 1.542 | 1.7154E-06 |
| 150 | 0.3523 | 0.000346 | 2.012 | 2.409E-07 | 0.352 | 0.001067 | 2.341 | 2.6637E-06 |
| 200 | 0.3518 | 0.000341 | 2.677 | 3.117E-07 | 0.3516 | 0.001046 | 3.101 | 3.3902E-06 |
| 250 | 0.3514 | 0.000336 | 3.353 | 3.786E-07 | 0.3512 | 0.00104 | 3.85 | 4.1675E-06 |
| 300 | 0.3509 | 0.000329 | 4.012 | 4.335E-07 | 0.3504 | 0.001032 | 4.639 | 4.9451E-06 |

From table above, we could tell that with 1000000 simulations, both methods give out really high efficiency. But the Quasi-Monte Carlo seems to give higher SE. To take a deep look at the efficiency, we do the plot as followed.

**Plot for Efficiency Comparison:**

Now, from this plot, we could easily tell that the Normal Monte Carlo method shows more stable SE, thus the dimension m has less effect on this method. Before, we’ve already discovered that the Normal Monte Carlo gives out smaller SE. To sum up, in this case, Normal Monte Carlo simulation with control variate would be more proper here and would give out an estimated price of 0.35.